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PARAMETER ESTIMATION BASED TYPE-II FUZZY LOGIC*

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Abstract. Regression analysis is an area of statistics that deals with the investigation of the dependence of a variable upon one or more variables. Recently, much research has studied fuzzy estimation. The fuzzy regression method can be used to obtain unknown parameters of regression models based fuzzy data. In this study, we will use the ANFIS for parameter estimation and propose an algorithm in case where the independent variables are fuzzy sets. These sets are type-II fuzzy sets because of characterized by a Gaussian membership function with fuzzy mean.

Keywords: type-II fuzzy logic, membership function, regression analysis, parameter estimation.

AMS Subject Classification 62A86.

1. Introduction

Fuzzy set theory was introduced by Lotfi A. Zadeh (1965) at a conference in United States. And the first serious step in this regard has been taken in an article published in 1965 by Lotfi A. Zadeh. In this study, fuzzy logic and fuzzy set theory is discussed in detail [2].

Over the last 30 years, studies on the theory of fuzzy sets have been conducted extensively. When there is an uncertainty about the membership functions fuzzy set is called as type-II fuzzy set. We can say that type-II fuzzy logic is a generalization of conventional fuzzy logic (type-I) in the sense that uncertainty is not only limited to the linguistic variables but also is present in the definition of the membership functions [2].

Studies on Type-II fuzzy clusters briefly summarized as follows:

Karnik and Mendel (1999), defined the uncertainty of the rules in a type-II fuzzy inference system that the rules are uncertain. They applied a type II fuzzy logic system to time varying channel equalization and this better performance than a type I fuzzy logic system and nearest neighbor classifier.

Türkşen (1999), proposed and discussed in fuzzy system development schema. For both the type-I and type-II fuzzy theory, they described the extraction of fuzzy sets and fuzzy rules with the application of an improved fuzzy clustering technique which is essentially an unsupervised learning of the fuzzy sets and rules from a given input-output data set.

Karnik and Mendel (2001), introduced the centroid and generalized centroid of a type-II fuzzy set and explained how are used to calculate them. Furthermore, they showed how to compute the centroid of interval and Gaussian type-II fuzzy sets.

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Karnik and Mendel (2002), discussed in set theoretic operations for type-II sets, properties of membership grades of type-II sets, and type-II relations and other compositions, and cartesian products under minimum and product t-norms.

Mendel and John (2002), defined a new representation theorem of type-II fuzzy sets and introduced formulas for the union, intersection, and complement for type-II fuzzy sets without having to use the Extension Principle by using this new representation.

Mendel (2007), examined questions, such as "What is a type-II fuzzy set", "What is it different from a type-I fuzzy set", "the importance of definition of type-II fuzzy sets", "How and why are type-II fuzzy sets used in rule-based systems" and "How are the detailed computations for an interval type-II fuzzy logic system" in study titled an introduction to type-II fuzzy sets and systems.

Mendel (2007), described the important advances that have been made during the past five years both general and interval type-II fuzzy sets and systems in the study titled "advance in type-II fuzzy sets and systems".

In the second part of the study will be definitions of type-II fuzzy logic method, in the third part described the structure of fuzzy inference system based on fuzzy adaptive network will be described. In the fourth part, an algorithm will be suggested to prediction of the unknown parameter of the regression model in the case of independent variable characterized by a normal membership function. A numerical application examining the work and validity of the suggested algorithm in the fifty part and in the last part a discussion and conclusion are provided.

2. Type-II fuzzy logic

Type-II fuzzy systems are consist of fuzzy if-then rules that are includes type-II fuzzy sets. Basically, a type-II fuzzy set is a set in which we also have uncertainty about the membership function. We can say that type-II fuzzy logic is a generalization of traditional fuzzy logic (type-I). Uncertainty is not on the limited to the linguistic variable but also is present in the definition of the membership function [1,2].

The concept of a type-II fuzzy set, was introduced by Zadeh in 1975 as an extension of concept of an ordinary fuzzy set. A type-II fuzzy set is characterized by a fuzzy membership function, the membership degree of each element of this set is in [0,1]. In this sense, differs from type-I fuzzy sets, because the degree of membership in the type-I fuzzy set is a crisp number in range of $[0, 1]$. Such sets can be used in situations where there is uncertainty about the membership degree and uncertainty in the shape of the membership function or in some of its parameters. The membership of an element in a set cannot determine as 0 or 1, type-I fuzzy set is used. Similarly, when the situation is so fuzzy that we have trouble determining the membership degree even as a crisp number in $[0, 1]$, fuzzy sets of type-II is used. In many real-world problems the exact form of the membership degree may not be identified. Consider the fuzzy set characterized by normal membership function with mean m and standard deviation can take values in $[\sigma_1, \sigma_2]$, the membership function is defined as

$$\mu(x) = \exp\left\{-\left[\frac{x-m}{\sigma}\right]^2\right\},\ \sigma \in [\sigma_1, \sigma_2]. \tag{1}$$

In this case, obtain a different membership function curve corresponding to each value of σ. In the same way, consider the fuzzy set characterized by normal membership function with standard deviation σ and mean can take values in $[m_1, m_2]$, the membership function is defined as;

$$\mu(x) = \exp\left\{-\left[\frac{x-m}{\sigma}\right]^2\right\},\ m \in [m_1, m_2] \tag{2}$$

and in both cases $\mu(x)$ is a fuzzy set.

In this study, the unknown parameters of regression model will be obtained in the event of the independent variables are fuzzy sets that characterized by normal membership function and mean of the membership function is a fuzzy number like as $m \in [m_1, m_2]$. Fuzzy adaptive network based fuzzy inference system will be used in order to obtain the unknown parameters of regression model [2].

3. Fuzzy adaptive network based fuzzy inference system

The Adaptive-Network Based Fuzzy Inference System (ANFIS) is a neural network architecture that can solve any function approximation problem. An adaptive network is a multilayer feed forward network in which each node performs a particular function on incoming signals as well as a set of parameters pertaining to this node and it has five layers. The formulas for the node functions may wary from node to node and the choice of each node function depends on the overall input-output function which the adaptive network is required to carry out.

A neural network enabling the use of a fuzzy inference system for fuzzy regression analaysis is known as an adaptive network. Used for obtaining a good approach to regression functions and formed via neurals and connections, such an adaptive network consist of five layers [4].

To illustrate how a fuzzy inference system can be represented by ANFIS, let us consider the following example. Suppose a data set has two-dimensional input $X = (x_1, x_2)$. For input $X = x_1$, there are two fuzzy sets "small" and "low" and for input x_2, two fuzzy set "large" and "high". In this case a fuzzy inference system contains the following for rules:

$$\begin{aligned}
R^1 &: If(x_1\ small\ and\ x_2\ low)\ then\ (Y = Y^1 = c_0^1 + c_1^1 x_1 + c_2^1 x_2)\\
R^2 &: If(x_1\ small\ and\ x_2\ high)\ then\ (Y = Y^2 = c_0^2 + c_1^2 x_1 + c_2^2 x_2)\\
R^3 &: If(x_1\ large\ and\ x_2\ low)\ then\ (Y = Y^3 = c_0^3 + c_1^3 x_1 + c_2^3 x_2)\\
R^4 &: If(x_1\ large\ and\ x_2\ high)\ then\ (Y = Y^4 = c_0^4 + c_1^4 x_1 + c_2^4 x_2)
\end{aligned}$$

There are two levels of nodes in layer one. The first level includes nodes "small" and "large" and the second level includes nodes "low" and "high". The output of the layer is the membership function based on the linguistic value of the input. Nodes in layer two output the products w^l $(l = 1, ..., 4)$ and the number of nodes in this layer is equal to combination of the nodes which are located in levels from layer one . Layer three performs a normalization of the output signals from layer two. Each node in layer four corresponds to the consequence of each fuzzy if-then rule. For example the first node in layer four includes $Y^1 = c_0^1 + c_1^1 x_1 + c_2^1 x_2$. Finally, the single node in layer five computes the overall output as the summation of all incoming signals from layer four [3].

The algorithm associated with the defined network structure of determining unknown regression parameter in the case of independent variables derived from a Normal distribution, is proposed as follows.

4. An algorithm for parameter estimation

The process of determining parameters of regression model begins with determining class numbers of independent variables and a priori parameters. The priori paremeters are characterized the distribution. In this work since the independent variables come from a gaussian distribution, we are interested in center (m) and spread (σ) In the case of independent variables come from gaussian distribution, the algorithm to obtain the unknown parameter of regression model is determined as follows.

Step 1: Class numbers related to the data set associated with the independent variables are determinated intuitively.

Step 2: An a priori parameter set is determined. Center and spread parameter are depend on level number of independent variables and its range.

Step 3: $\overline{w}^L$ weights are counted, which are used to form matrix B to be used in determining posterior parameter set. $\overline{w}^L$ weights are outputs from the third layer of the adaptive network and calculated using the membership function of Normal distribution. When independent variable numbers are indicated with p and if the fuzzy class number associated with each variable is indicated by $l_i; i = 1, ..., p$ the fuzzy rule number is indicated by $L = \prod_{i=1}^{p} l_i$. The hth node in the first layer is adaptive, and is defined as

$$f_{1,h} = \mu_{F_h}(x_i). \tag{3}$$

Where fuzzy cluster related to fuzzy rules are indicated with $F_1, F_2, ..., F_h$ and μ_h is the membership function related to F_h. Different membership function can be defined for F_h. Here membership functions are defined as

$$\mu_{F_h}(x_i) = \exp\left[-\left(\frac{x_i - m_h}{\sigma_h}\right)^2\right]. \tag{4}$$

Here, $\{m_h, \sigma_h\}$ is priori parameter set suitable for gaussin distribution and m is a fuzzy parameter and takes values in the range of $m \in [m_1, m_2]$.

Membership degrees of independent variables are determinated by defined membership function which is given in Eq. (4.2). w^L weights are obtained from the multiplication of these membership degrees and defined as

$$w^L = \mu_{F_L}(x_i)\,\mu_{F_L}(x_j). \tag{5}$$

$\overline{w}^L$ weights are normalization of the w^L and determinated by

$$\overline{w}^L = \frac{w^L}{\sum_{L=1}^{m} w^L}. \tag{6}$$

Step 4: When the one of the priori parameter m is a fuzzy number, the posterior parameter set c_i^L which is the unknown coefficients of regression model obtained as a fuzzy number shape of $c_i^L = \left(a_i^L,\ b_i^L\right)$ $(i = l, ..., p)$. Under this condition, the equalty $Z = \left(B^T B\right)^{-1} B^T Y$ is used for determining the a posteriori parameter set. Here, B and Y defined as

$$B = \begin{bmatrix} \overline{w}_1^1 & \cdots & \overline{w}_1^m, & \overline{w}_1^1 x_{11}, & \cdots & \overline{w}_1^m x_{11}, & \cdots & ,\overline{w}_1^1 x_{p1} & \cdots & \overline{w}_1^m x_{p1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \overline{w}_k^l x_{jk} & \ddots & \vdots & \ddots & \vdots \\ \overline{w}_n^1 & \cdots & \overline{w}_n^m, & \overline{w}_n^1 x_{1n} & \cdots & \overline{w}_n^m x_{1n}, & \cdots & ,\overline{w}_n^1 x_{pn} & \cdots & \overline{w}_n^m x_{pn} \end{bmatrix},$$

$Y = [y_1, y_2, \cdots, y_n]^T$.

Step 5: By using the posteriori parameter set $c_i^L = \left(a_i^L,\ b_i^L\right)$ obtained in Step 4, the regression model indicated by $Y^L = c_0^L + c_1^L x_1 + c_2^L x_2 + ... + c_p^L x_p$. Setting out from the models and weights specified in Step 3, the prediction values are obtained using

$$\hat{Y} = \sum_{L=1}^{m} \overline{w}^L Y^L. \tag{7}$$

Step 6: Error related to the model is measured as

$$\varepsilon = \frac{\sum_{k=1}^{n}(Y_k - \hat{Y}_k)^2}{n}. \tag{8}$$

If $\varepsilon < \phi$, then the posteriori parameter has been obtained as parameters of regression models to be formed, the process is determinated. If $\varepsilon \geq \phi$, then Step 6 begins. Here ϕ is a law stable value determinated by decision maker.

Step 7: Central priori parameters specified in Step 1, are updated with $v_i^{'} = v_i \pm t$ in a way that it increases from the lowest value to the highest and decreases from the highest value to the lowest. Here, t is size of step;

$$t = \frac{\max(x_{ji}) - \min(x_{ji})}{a} \; ; j = 1, 2, ..., n; \quad i = 1, 2, ..., p \tag{9}$$

and a is stable value which is determinant of size of step and therefore iteration number.

Step 8: Predictions for each priori parameter obtained by change and error criterion related to these predictions are counted with

$$\varepsilon_k = Y_k - \widehat{Y}_k. \tag{10}$$

Here; Y_k is k. predicted outcome and $\widehat{Y}_k$ is k. network output of input vector.

The lowest of error criterion is defined. Priori parameters giving the lowest error specified, and prediction obtained via the models related to these parameters is taken as output.

5. Numerical example

Data set used in the application is selected from the literature and includes two independent variables and one dependent variable. This set is located in the Table 1.

Applying least squares, the estimated regression model yields

$\widehat{Y} = 0.159 - 0.0515X_1 + 0.2650X_2$

and the algorithm proposed in Section four was counted with a program written in MATLAB. From the program, the regression models based fuzzy inference systems are as follows

$$\begin{aligned}
\widehat{Y}_1 &= (3.4751; 0.3987) + (1.3074; 0.1214)X_1 - (1,5979; 0.1681)X_2, \\
\widehat{Y}_2 &= (126.3823; 11.9832) - (5,5898; 0.6504)X_1 - (0.0736; 0.0081)X_2, \\
\widehat{Y}_3 &= (-220.4047; 21.3941) + (4.7533; 0.4659)X_1 - (5.6617; 0.5765)X_2, \\
\widehat{Y}_4 &= (185.3528; 19.5434) - (8.3921; 0.9324)X_1 + (3.1306; 0.2987)X_2.
\end{aligned}$$

6. Conclusion

The independent variables are comes from a normal distribution, and regression models are formed using membership functions that are appropriate to the normal distribution. Since he central parameter m located in normal membership function is fuzzy parameter and takes values in the range $m \in [m_1, m_2]$ the unknown parameters of regression model are obtained as fuzzy numbers. The prediction values obtain from adaptive network and the prediction values obtained from least square estimates are compared. According to the indicated error criterion, the errors related to the predictions that are obtained from the network are less than errors obtained from the least square estimates.

Table 1. Prediction and error values for data set

X_1	X_2	Y	$\widehat{Y}_{(Network)_i}$	$e_{(Network)_i}$	$\widehat{Y}_{(LSE)_i}$	$e_{(LSE)_i}$
24.7000	15.0000	2.6500	(2.6291;0.2789)	-0.0209	2.8611	-0.2111
24.8000	17.0000	2.6300	(3.0277;0.2941)	0.3977	3.3859	-0.7559
26.5000	19.4000	4.9500	(4.2411;0.4314)	-0.7089	3.9343	1.0157
29.6000	20.1000	4.4900	(4.3820;0.4284)	-0.1080	3.9600	0.5300
25.7000	19.5000	3.1700	(3.7564;0.3562)	0.5864	4.0020	-0.8320
25.0000	20.1000	3.8800	(3.6307;0.3765)	-0.2493	4.1971	-0.3171
21.6000	16.3000	3.9000	(4.0228;0.3925)	0.1228	3.3653	0.5347
24.7000	18.0000	3.5100	(3.1563;0.2812)	-0.3537	3.6561	-0.1461
25.9000	18.3000	3.9000	(3.7009;0.3689)	-0.1991	3.6737	0.2263
25.6000	18.7000	3.4700	(3.5871;0.3681)	0.1171	3.7952	-0.3252
27.9000	21.9000	5.5300	(4.6815;0.4585)	-0.8485	4.5246	1.0054
25.8000	20.0000	3.4800	(3.8893;0.3965)	0.4093	4.1294	-0.6494
26.2000	20.2000	4.3500	(4.1272;0.4015)	-0.2228	4.1618	0.1882
24.8000	21.5000	4.3800	(4.1821;0.4712)	-0.1979	4.5784	-0.1984
27.7000	20.6000	4.3900	(4.7187;0.4845)	0.3287	4.1905	0.1995
26.2000	22.0000	5.0200	(4.7062;0.4712)	-0.3138	4.6388	0.3812
23.9000	22.0000	4.7700	(4.7769;0.4856)	0.0069	4.7573	0.0127
28.1000	23.4000	5.3600	(5.3601;0.5402)	0.0001	4.9118	0.4482
23.0000	18.5000	3.8500	(3.5573;0.3672))	-0.2927	3.8762	-0.0262
26.0000	16.4000	2.9400	(3.0494;0.2895)	0.1094	3.1651	-0.2251
23.6000	21.0000	4.2800	(4.3751;0.4465)	0.0951	4.5077	-0.2277
22.4000	15.0000	4.1400	(4.1108;0.3906)	-0.0292	2.9796	1.1604
22.6000	19.4000	4.3100	(4.2770;0.4126)	-0.0330	4.1353	0.1747
23.4000	20.3000	4.1300	(4.1180;0.4248)	-0.0120	4.3326	-0.2026
27.5000	22.0000	3.6400	(4.8504;0.5012)	1.2104	4.5718	-0.9318
32.0000	19.4000	3.4200	(3.4581;0.3525)	0.0381	3.6509	-0.2309
25.2000	20.2000	3.2100	(3.7052;0.3801)	0.4952	4.2133	-1.0033
24.7000	14.6000	2.6200	(2.5190;0.2499)	-0.1010	2.7551	-0.1351
23.4000	21.7000	4.8600	(4.9977;0.5121)	0.1377	4.7035	0.1565
26.2000	21.8000	4.9700	(4.6058;0.4721)	-0.3642	4.5858	0.3842
ERROR			$\varepsilon_{Network} = 0.1479$		$\varepsilon_{LSE} = 0.2906$	

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References

[1] Aisbett, J., Rickart, J.T., Morgenthaler, D., (2011), Multivariate modeling and type-2 fuzzy sets, Fuzzy Sets and Systems, 163, pp.78-95.

[2] Castillo, O., Melin, P., (2008), Type-2 Fuzzy Logic: Theory and Applications, Sipringer.

[3] Cheng, C.B., Lee, E.S., (2001), Switching regression analysis by fuzzy adaptive network, Europen Journal of Operational Research, 128, pp.647-668.

[4] Hisao, I., Manabu, N., (2001), Fuzzy regression usin asymmetric fuzzy coefficients and fuzzied neural networks, Fuzzy Sets and Systems, 119, pp.273-290.

[5] Karnik, N.K., Mendel, J.M., (1999), Type-2 fuzzy logic systems, IEEE Transaction on Fuzzy Systems, 7, pp.643-658.

[6] Karnik, N.K., Mendel, J.M., (2001), Centroid of a type-2 fuzzy set, Information Sciences, 132, pp.195-220.

[7] Karnik, N.K., Mendel, J.M., (2002), Operations on type-2 fuzzy sets, Fuzzy Sets and Systems, 122, pp.327-348.

[8] Mendel, J.M., John, R.I.B., (2002), Type-2 fuzzy sets made simple, IEEE Transaction on Fuzzy Systems, 10, pp.117-127.

[9] Mendel, J.M., (2007), Type-2 fuzzy sets and systems: an overview, IEEE Computational Intelligence Magazine Fabuary, pp.21-29.

[10] Mendel, J.M., (2007), Advances in type-2 fuzzy sets and systems, Information Sciences, 177, pp.84-110.

[11] Türkşen, I.B., (1999), Type I and type II fuzzy systems modelling, Fuzzy Sets and Systems, 106, pp.11-34.

[12] Zadeh, L.A., (1965), Fuzzy sets, Information Science, 8, pp.338-353.

[13] Zadeh, L.A., (1975), The concept of a linguistic variable and its application to approximate reasoning- I, Information Science, 8, pp.199-249.

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